

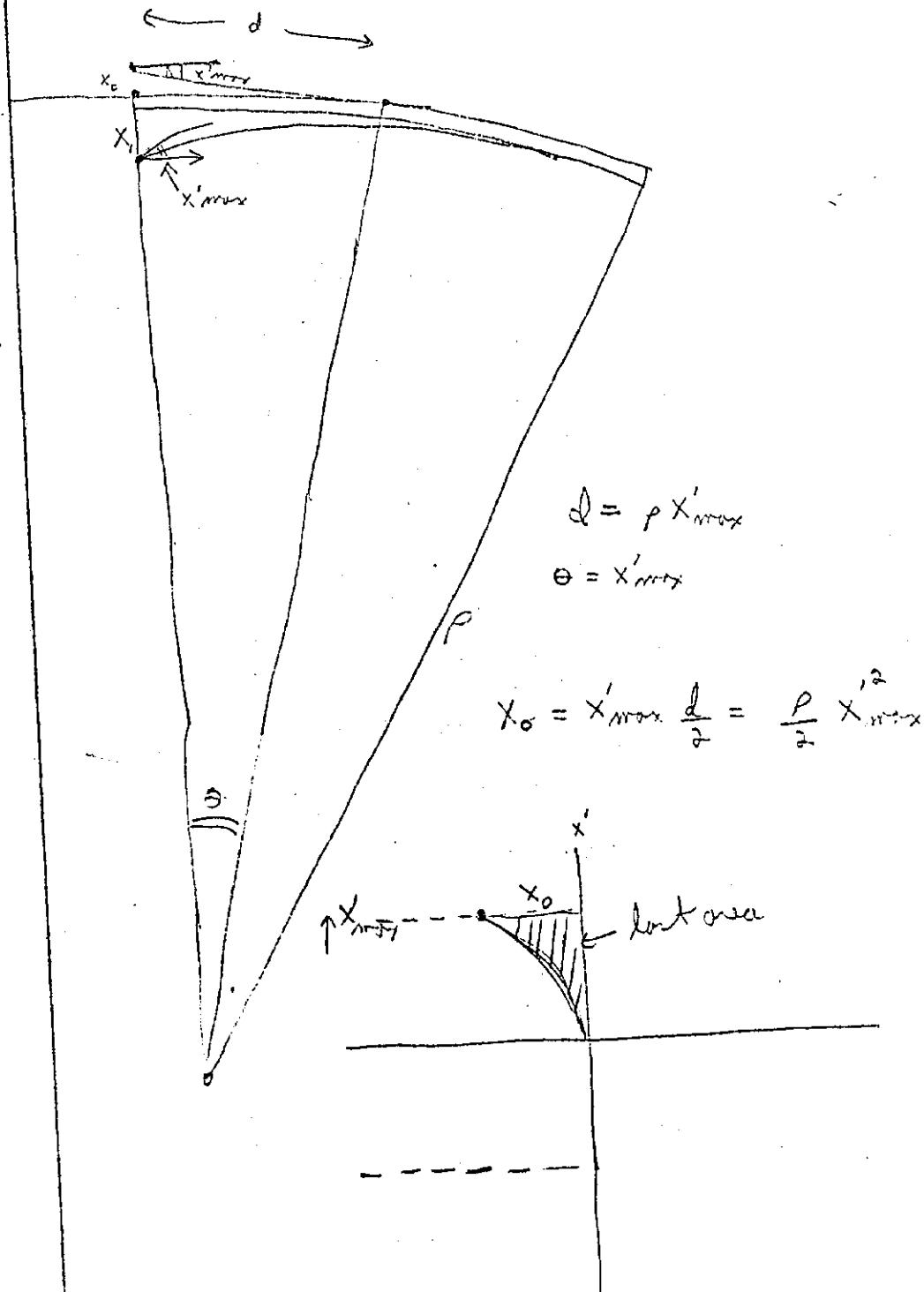
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## Effective width of septa:

Consider a beam of angular divergence  $\pm x'_{\max}$ .

We assume this is incident on a thin septum which is curved according to the radius of curvature of particles in the magnet.



The phase space area in this volume is given by:

$$\int_0^{x_{\max}} dx' x_0 = \frac{\rho}{2} \int_0^{x_{\max}} x'^2 dx' = \frac{\rho x'^3}{6}$$

Now this area will be lost on both sides, so we get:

$$\text{total loss} = \frac{\rho x'^3}{3}$$

Now if we want to convert this to a thickness, this loss can be equated to a rectangular area of:

$$2x'_{\max} t_{\text{eff}} = \frac{\rho x'^3}{3}$$

$$\therefore t_{\text{eff}} = \frac{\rho x'^2}{6}$$

Now the divergence of the external beam can be calculated. We assume .1 cm-rrad for the emittance of the beam in the machine. We take the extracted beam to have  $\frac{1}{4}$  of this value, and to be h cm wide, say.

$$2hx'_{\max} = \frac{1}{4} \cdot 10^{-4} \text{ cm-rradian}$$

$$\therefore x'_{\max} = \frac{1}{8h} \times 10^{-4} \text{ cm-rradian}$$

The complete width of the septum then is given by  $T = t_s + t_{\text{eff}}$

The extraction efficiency is given approximately by the relation:

$$\eta = 1 - \frac{T}{h}$$

Now  $t_s$  and  $\rho$  are related by the current density in the copper i.e.

$$B\rho = p \quad \rho = \frac{p}{B}$$

$$\text{Also } B = j t_s$$

If we now look at  $T_h$  we get the expression:

$$1 - \eta = \frac{1}{h} \left( t_s + \frac{\rho x'^2}{6} \right)$$

$$\text{using } \rho = \frac{p}{j t_s}$$

$$1 - \eta = \frac{1}{h} \left( t_s + \frac{p x'^2}{6 j t_s} \right)$$

Since we want to minimize this expression, we find the optimum thickness given by:

$$|t_s|_{opt} = x'^{max} \sqrt{\frac{p}{6 j}}$$

The optimum given:

$$|1 - \eta|_{opt} = \frac{2 x'^{max}}{h} \sqrt{\frac{p}{6 j}}$$

The momentum at 200 Gev/c =  $6.67 \times 10^8$  gauss-cm  
 Suppose we take a value of  $10^4$  gauss/cm for  $j$ .

$$\sqrt{\frac{P}{6j}} \approx 100 \text{ cm}$$

Now  $x'_{\max}$  is  $\approx 10^{-5}$  radian.

$$\therefore |1 - \eta| \underset{\text{opt}}{\approx} 10^{-3}$$

this corresponds to an  $\approx$  1 mil septum, and a field of about 25 gauss. ( $\approx$  25 amps in a half-inch high strip). This also implies an effective thickness of septa of about 1 mil also. Therefore we see that for thicker (more realistic?) septa, the effective thickness can be made vanishingly small. In fact, the problem will be dominated by consideration of alignment tolerances, etc.

The point of this exercise is just to point out that achievable current densities in copper are not what will limit the extraction efficiency, above perhaps .1%.

We treat the problem of a beam grazing a thin septum. In a theoretically optimum system the septum width would increase to keep the septa well near the grazing particle. Keeping constant current density  $j$  in the copper, this means that the field will increase with increasing displacement from the axis. The equation for the "ideal" curve can be obtained from the matrix for a defocusing quadrupole:

$$\begin{bmatrix} X(s) \\ X'(s) \end{bmatrix} = \begin{bmatrix} \cosh ks & \frac{1}{k} \sinh ks \\ k \sinh ks & \cosh ks \end{bmatrix} \begin{bmatrix} X_0 \\ 0 \end{bmatrix}$$

where  $k^2 = \frac{G}{p}$ , and  $G = j \therefore k = \sqrt{\frac{j}{p}}$

then  $X(s) = X_0 \cosh ks \quad X_0 = t_s$

Now  $s$  and  $X(s)$  are given by the machine parameter. In fact, since 10 kg is about the maximum field we will consider here, the above relation will only hold up to  $X(s) \approx 1\text{cm}$ , typically and we will consider a uniform scaling of curvature after that.

For the NAL design, therefore, we get  $s = 4000\text{ cm}$ , taking  $k = \sqrt{\frac{1}{6.67} \times 10^{-4}} = .387 \times 10^{-2}\text{ cm}^{-1}$   
 $ks = 15.5$

$$\therefore X_0 = \frac{1\text{ cm}}{\cosh 15.5} = .4 \times 10^{-6}\text{ cm}$$

and furthermore our calculation of optimum septum thickness give a value of about 1 mil. Therefore it is a more interesting problem to assume a current density, and an  $x_0$ , and find  $s$ :

Let's take the case of 1 mil  $\Rightarrow .0025 \text{ cm}$ .  
then we have:

$$\cosh ks = 400$$

$$ks = 6.7$$

$$\therefore s = \frac{6.7}{.387} \times 10^3 = 1700 \text{ cm} \Rightarrow 17 \text{ meters.}$$

Since 40 meters are available for this race, it seems plausible that such a system could in fact be built.  
We will show such a design in another note.

Now let's look at the angle of bend which we obtain for this system:

$$x'(s) = kx_0 \sinh ks$$

$$\text{For } ks = 6.7, \sinh \approx \cosh = \frac{x}{x_0}$$

$$\therefore x' = kx \quad \text{when we took } x = 1 \text{ cm.}$$

$$\text{this gives } x' = 3.87 \text{ mr.}$$

It is interesting to see how these calculations go as a function of the current density:

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If we halve the current density, we then continue the calculation until  $x = 2 \text{ cm}$ . (10 kg again)

then  $\cosh ks = 800$

$$ks = 7.38 \quad k = \frac{387}{\sqrt{2}} \times 10^{-3} = .274$$

$$s = 27 \text{ meters.}$$

Note that halving the current density by a factor of two (Unibeam) will double the  $t_{eff}$  of the system giving a lower extraction efficiency by a factor of  $\frac{2}{3}$ .

The results of these two notes point up the following facts.

- 1.) Obtainable current densities in copper are high enough to attain  $10^{-3}$  extraction loss.
- 2.) These same current densities are high enough to extract the beam from the long S.S. without the use of septa in any other S.S.
- 3.) Mechanical tolerances, reliability, ease of operation, and fabrication skill will be the dominating restrictions on the design. i.e., a 10% efficient extraction system has few and simpler components, but creates a worse radiation hazard. On the other hand, a 1% system has a lot of components, but lower radiation levels. With extraction efficiency in excess of 90%, there is negligible gain in the physics program by increasing the efficiency. Any gain above this should be in the direction of reliability and performance.

The above considerations point up the desirability of designing a flexible system. Such a system will be described in a later note.